CERTAIN INVESTIGATIONS RELATING TO BLOOD FLOW IN MICRO CIRCULATORY SYSTEM

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Certificate

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Preface

The present thesis entitled 'Certain Investigations relating to blood flow in micro circulatory system' is a record of genuine research work carried out by me under the supervision and guidance of Dr. H.S. Shukla (Supervisor), Principal, Pt. J.N. College, Banda.

The purpose of the thesis is to study some problems on steady and pulsatile laminar viscous flow of blood in micro circulatory system i.e. the flow of blood in those vessels whose diameters are less than 500 μm ($1\mu m = 10^{-6}$ m).

The effects of peripheral layer, yield stress and other rheological parameters on blood flow have been analysed. We have studied the problems on dispersion process in blood flow and effects of mild stenosis on blood flow through narrow vessels. In the analysis it has been assumed that the radius of the vessel is constant and there is no slip velocity at the wall.

This thesis consists of six chapters. Chapter I contains the general introduction of the subject (viz. Circulatory system, rheology of blood, geometrical aspect of vessels etc.) and a brief account of the relevant literature.

In Chapter II, we have discussed steady shear flow of micropolar fluid through a channel of large cross-section. In the present chapter we have proposed to analyse the fully developed steady shear flow of micro polar fluid between two parallel plates. The polarity arising due to the presence of buoyant corpuscles in the fluid leads to give a theoretical insight of the blood flow through large arteries. The microrotional field depends on the length ratio parameter. Microrotation effects and microrotational inertia are considered in

this case. Exact steady solution for velocity and microrotation subject to the strong and weak limit of influence of surface on microrotation are obtained. The parameters characterising the ratio of vortex viscosity to sheer viscosity and length ratio which do not appear in a classical Newtonian fluid are introduced and its effects on velocity and microrotation are discussed. The results are discussed with the help of Tables.

In Chapter III we have discussed steady flow of blood in Narrow tube through micropola fluid model with slip at the wall. In the present chapter we have studied the poiseville flow of micropolar fluid as a model of blood flow in 40 μm diameter tube and for 40% RBC concentration.

At the boundary, we have introduced a slip in the axial velocity and a partial rotation depending upon the wall effect paramters and fluid vorticity. From the analysis we have observed that the introduction of slip reduces the apparent viscosity of the suspension and increases its axial velocity whereas it has no effect on particle rotation. Effect of $\bar{S}$ ($0 \leq \bar{S} \leq 1$; a parameter depending on $\bar{S}$ and concentration) on different flow parameters are seen and by comparing the results is in the experimental values we have tried to estimate the value of S. Results have been discussed with the help of tables.

In Chapter IV, a mathematical analysis for the dispersion of soluble matters in blood flow has been carried out. In present problem we have prepared the flow model with casson fluid in the core region surrounded by a Newtonian plasma layer near the wall. Taylor's limiting condition and Fick's Law of diffusion are used for finding the solution of problem. The effective dispersion co-efficient with which the solute disperses across a plane moving with mean speed of the medium is found to be decreased with respect to the yield stress and molecular diffusion co-efficient where as a reciprocal behaviour is observed with respect to the viscosity of the casson fluid. The results are discussed with the help of tables.

In Chapter V, we have discussed the effects of wall layer thickness on

blood rheology. From several studies it has been established that for blood flowing in narrow tubes (500 μm to 50 μm) a cell free plasma zone exists near the wall and the extent of this depends on many rheological parameters. From the analysis presented here, we see that apparent viscosity of blood decreases as the thickness of plasma layer increases.

In Chapter VI, a theoretical model of steady blood flow in narrow vessel have been considered. In present analysis we have assumed that fluid in core region satisfy casson equation and in marginal layer region satisfy Newtonian equation. Apparent viscosity of blood has been determined as a function of yield stress, vessel diameter and peripheral layer (PPL) thickness.

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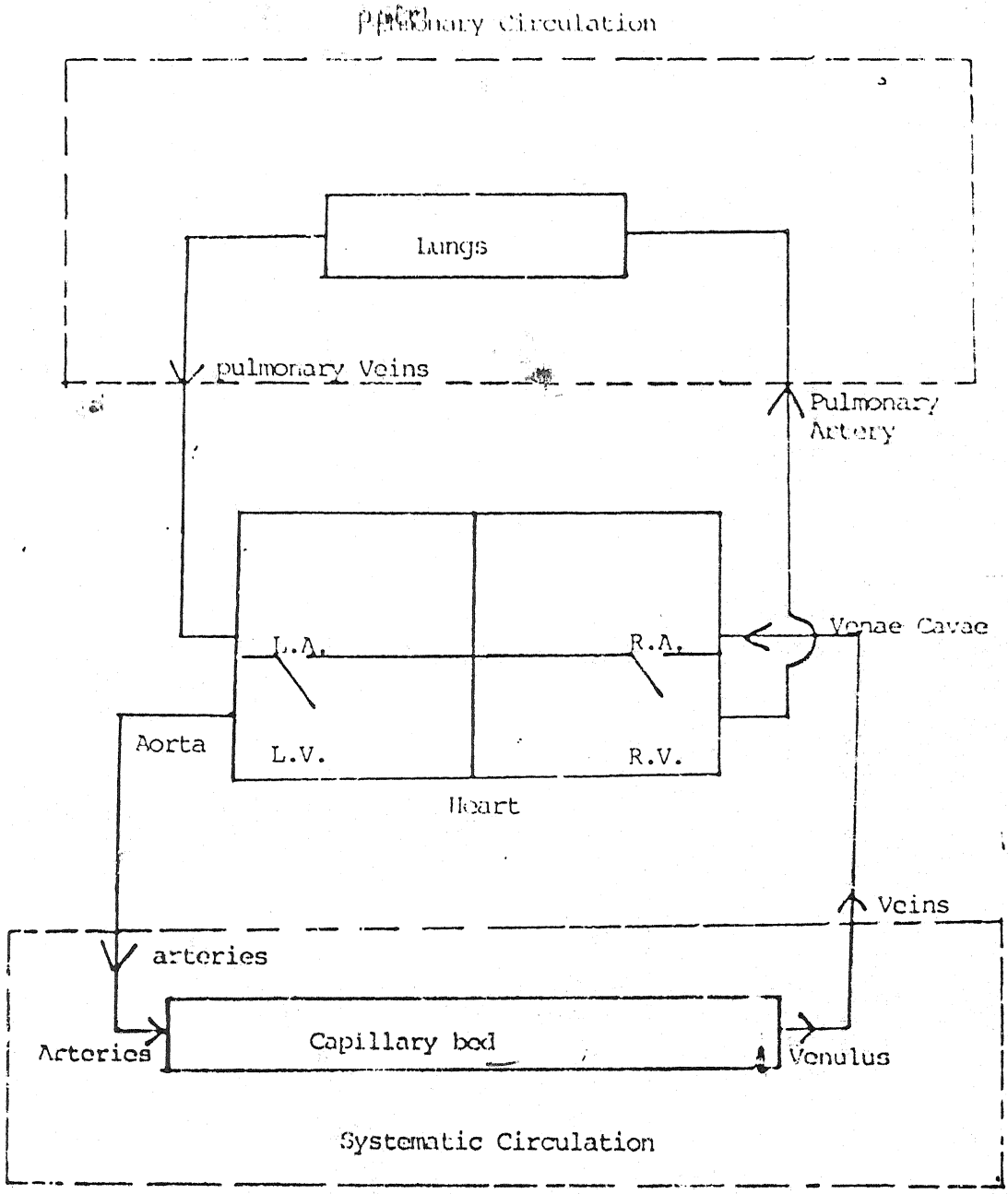

Fig.1: Human Circulatory System.

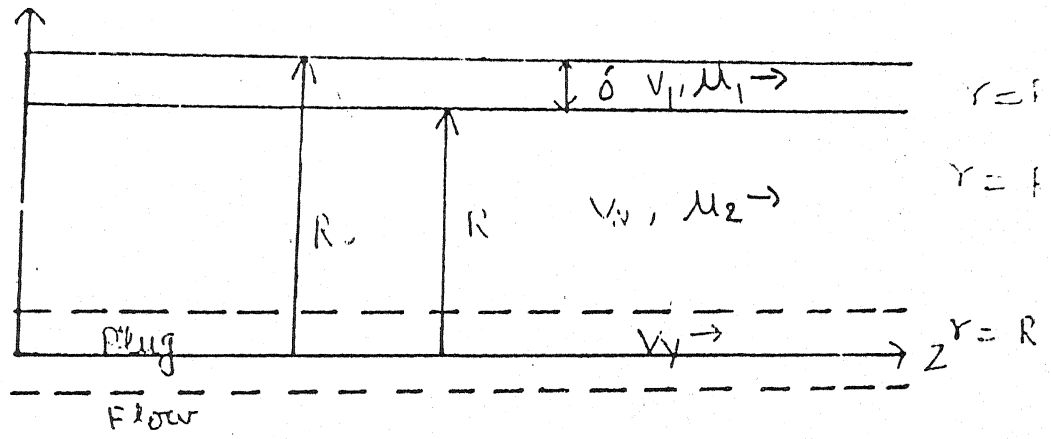

Fig 2.1 Geometery of the Vessel

GENERAL INTRODUCTION

1.1 INTRODUCTION

Bioengineering is the field which has been developed recently by cross fertilization of the engineering and biological sciences, so that both are going to be utilized more effectively for the benefit of mankind.

Biomechanics is an important branch of bioengineering. This is the study of the mechanical aspects of the structure and function of biological systems. Biofluid mechanics, an important branch of biomechanics, is the study wherein the principles of fluid mechanics are applied to understand the problems in biology. Fluid mechanics is essentially the science concerning properties and motion of fluids. In the present Analysis we shall concentrate only on the application of fluid mechanics to study the blood rheology and blood flow in human circulation.

'Blood', the red fluid of the blood vessels with which we are all familiar, is the transport medium of the body. It is the medium by which all living tissues are related to their external environment, i.e. from the outside world. Blood is pumped to and from between tissues and elementary canal to take up oxygen and nourishment respectively, and it helps

in returning the products of oxidation and metabolism in the tissues to the outside world by the lungs, the skin and kidneys. It is also the medium by which growth and repair substances are transported to the tissues and by which the various controlling glands of the body can distribute their chemical messengers.

Blood is some what viscid fluid. In man and in all other vertebrate animals with the exception of two (the amphioxus and leptocephalus), it is red in colour. It consists of a continuous yellowish aqueous phase, the plasma, in which formed elements are suspended. The formed elements consist of red blood cells (erythrocytes), white blood cells (leukocytes) and platelets (thrombocytes). Plasma is made up of water (92%) and contains traces of inorganic and organic salts. The inorganic and small organic molecules contribute little to plasma viscosity which is primarily dependent on the plasma proteins. If blood is allowed to clot and the solid material be removed, the remaining fluid is called serum. This has essentially the same composition as plasma except that fibrinogen and same of the clotting factors have been removed. About 7%, by weight, of plasma is protein mainely albumin, globulin and fibrinogen, which have molecular weight ranging from 44,000 to 10,00000. About half of the protein mass is albumin. The significance

of plasma protein is its multi form natures mentioned below :-

i) Plasma proteins are responsible for osmotic pressure, the level of which is important for regulating water exchange between blood and tissues.

ii) They possess buffer properties and maintain the acid-base equilibrium of blood .

iii) They produce a definite viscosity of plasma which is important in maintaining blood pressure.

iv) They promote stabilization of the blood by providing conditions that prevent sedimentation of erythrocytes.

v) They play an important role in coagulation.

vi) Plasma proteins are important factors in immunity.

If blood, to which an anticoagulant has been added, is poured into a test tube and centrifuged, the corpuscles tend to settle at the bottom and the blood is divided into two layers, namely, a red layer lower consisting of the formed elements and a transparent colourless or slightly yellowish upper layer consisting of plasma. The leukocytes form of thin white film between the erythrocytes and the plasma since thier specific gravity is less than that of the erythrocytes.

The red cells surface carrier a net negative charge, but in stationary blood, cells interact with

each other to form aggregates. The aggregates commonly consist of 6 to 10 cells stacked face to face and such a clustor of cells is called a rouleaux. Secondary aggregation of rouleaux also occurs building up a complex three dimensional structure. When blood is sheared, these rouleaux breakup and at sufficiently high shear rates the cell exists as an individual. Blood flows in the form of different laminar containing different types of cells. Considering these factors, the rheological properties of blood might be expected to be rather complex.

The red cells out number the white cells and platelets by approximately 5000 : 1 and 12 : 1 respectively. In normal blood, cellular components other than red cells compose only about 3% of the total cell volume and usually have only a small effect on blood viscosity. A suspension of leukocytes shows a higher viscosity of white cell in a volume fraction. Steinberg and Charm [140] Further, white cells may significantly increase blood viscosity in diseases where there is large increase in leukocytes concentration (Dintenfass [38]).

The red blood cells are about 97% of the total cell volume in the blood. Consequently, the removal of white blood cells and platelets does not measurably modify the experimently determined flow properties of these suspensions. The concentration of most red blood

cells in suspension is generally reported as the suspension volume fraction occupied by the red blood cells, called hematocrit. The hematocrit is normally about 42-45% of volume. The erythrocytes consist of a thin flexible unstretchable membrane with an interior filled with saturated hemoglobin solution (viscosity 6.0 centipoise). The membrane is highly deformable if the change in the surface area is small, but becomes very much stiffer if the deformation produces a large change in the area of the cell membrane. The red blood cells have the shape of a biconcave disc which can deform, however, into a bullet shaped entity during passage through small capillaries. The cell has a large surface area relative to its volume.

Blood remains fluid in the blood vessels throughout its life but rapidly becomes solid when shed. These two qualities are essential for the preservation of life. The maintenance of fluidity is necessary for the circulation of blood, whilst the solidification of the red blood provides an indispansable defence against excessive bleeding from wounds. The coagulation of the blood is due to formation of a jelly by the deposition of protein material called 'fibrin' and it is the formation of this body that the fundamental change in blood clotting occures. To prevent this from occuring, various anticoagulents are added to the blood when it is drawn from the animal.

The effects of an anticoagulent on rheological properties of blood are difficult to as certain because of the difficulty in making reliable rheological measurements in the short time available between the blood drawing and the blood clotting. The effect of anti coagulents on blood viscosity appears to be small (Charm and Kurland [26]; Cokelet et al [34]).

Human Circulatory System :

The circulatory system of blood consists of complex functions (Respiratory, Nutritive exctretory, protective and regulatory) in the human body. The heart that provides the energy for the circulation has four compartment (i.e. left and right atrium and left and right ventricles) interconnected to each other by one way valves. Blood comming from body tissues enters the right atrium through the venul caval contraction of right atrium forces blood past the Tricuspid valve into the right ventricle. From this point, there begin two subdividsions, viz. the pulmonar and the systemic circulations. As the term shows, the former services the lung and latter the various systems of the body. The blood vessel has internal diameter in the range of 2.5cm in. aorta to about 4 microns in capillary (Keela and Neil [71].

Blood is pumped from the heart into the aorta from where it goes to the circulatory system consisting

of about 40 large arteries, 1600 main artery branches, 1800 terminal branches, 4,00,00,000 arterioles, 1,20,00,00,000 capillaries, 8,00,00,000 venules, 1,800 terminal veins, 600 main venous branches, 40 large venous branches, 40 large veins and then returns to the heart through the venule caval

It is estimated that the heart muscles itself consume about 10% of the energy required sustain life. Only some energy goes into the mechanical work of pumping blood. The heart beats about 70 times in one minute in an average person at rest. Blood flow in smaller blood vessel approaches steady flow condition otherwise pulsatile. The pressure in the aorta rises rapidly to its maximum (systolic) value of about 120 mm/Kg. The requirement of the circulation is the supply of oxygen required for metabolic pressures. In an average man at rest, the O_2 requires 200 ml/min. Under physical stresses, the need may rise to 5 lit./min. Blood has a capacity of 200 ml of oxygen per litre.; The efficiency of transfer of O_2 from blood to the cells is such that for an average man it is necessary for the heart to circulate 5 to 6 litres of blood per minute or about the 5 times the metabolic requirement of oxygen. The 'metabolic requirements' are the energy requirements for biological functions.

In fluid mechanics, the circulatory system is classified into two parts, viz. macrocirculation and

microcirculation.

(a) Macrocirculation :

The flow of blood in vessels of diameter greater than 500 μm (such flow occures in aorta) is called macro circulation. The flow is characterized by high Reynolds number, defined as the ratio of inertial to viscous forces. Turbulent flow can occur in blood vessels for the Reynolds number greater than 2300. The governing equations for analysing of such flow conditions should include inertial effects, effect of curvature of blood vessel pulsatile flow and the distensibility of the vessel wall. Blood can be taken as homogeneous and continuum fluid when vessel diameter is large in comparision to the dimensions of red blood cells. Pulsatile flow effects and pressure wave propagations have been studied in macrocirculation by a number of research workers, Skalak [133] and Taylor [151]. The flow disturbances at bifurcations, bends etc. and their effects on pathological states are studied by Patel et al [110]

(b) Microcirculation :

When the diameter of blood vessel is less than 500 μm (i.e. arterioles, capillaries, venules), the circulation is called microcirculation. This

is responsible for 80% pressure drop in circulatory system. In capillary bed the transfer of nutrients to and removal of wastes form the living cells of the body is a part of microcirculation. The flow is characterized by very low Reynolds numbers (Re <1). But we can not neglect the size of the red blood cell, compared to blood vessels size, it becomes necessary to treat the flow as two phase flow (non homogeneous). This two phase nature of flow is responsible for anomalous behaviour in blood flow through narrow tubes. The diameters of the capillaries ranges are approximately 3.5 μm to 10 μm. The problem is generally analysed by either high speed photography or by model studies. (Branemark [10], Gross and Aroesty [54]

Here we are interested in microcirculatory system . Table 1.1 gives basic information related to the properties of the vessels through which the circulating blood passes. When blood flow through capillaries of internal diameter equal to that of the red cell, the flow pattern is entirely different from that in the large blood vessels. The red blood squeezes through the capillaries one after the other, with plasma trapped in between them. Hegen-Poiseuille law, axial accumulation of cells and the parabolic velocity distribution have not relevance to capillary

flow. The flow pattern, developed when a train of cells follows each other through such a narrow tube, has been termed as bolus flow. Between the rear end of one cell and the leading end of the next, circulation patterns are set up in the axial core of plasma. The effect of this flow is that abolus of suspending liquid between two cells is carried down in the tube at the velocity of the cells.

1.3 RHEOLOGY OF BLOOD:

Blood behave like a time dependent non-Newtorian fluid and the basic rheological property of blood is its viscosity. The viscosity of whole blood is about 4.0 centipose and of plasma is about 1.2 centipose (Merril [90] at 37 °C. The specific gravity of whole blood is 1.05 to 1.06 (Cokelet [34]) and that of plasma is about 1.03. Thus the red blood cells tend to sediment slowly in plasma. The increased viscosity relative to water is produced mainly by the presence of plasma proteins. Both, the molecular shape and concentration of the protein are important. Fibrinogens which has an elongated molecule has a marked influence; although forming less than 5% of the total plasma protein forming less than 5% of the total plasma proteins it is responsible for about 20% of the plasma viscosity elevations.

1.3(a) The Viscosity of Blood:

It is an important factor is determing the local pressure variation through the cardio-vascular system, which

in turn influences the localflow rates through each section of the vascular network. The clinical importance of blood viscosity as a parameter lies in its sensitivity to small variations in composition. One can often diagnose pathological states by determining a change in blood viscosity.

There are several rheological parameters (e.g. plasma, blood cells, hematocrit etc.) which effect the blood viscosity. The viscosity of plasma increases with its protein concentration. But some proteins have different influences on plasma viscosity depending on their shape and size. The influence of Fibrinogen on plasma viscosity can be seen in the difference between plasma and serum viscosities. Serum usually has a viscosity which is 20% less than that of plasma. The influence of globulins on viscosity is illustrated the disease macroglobulienemia (Soner [135]. Albumin, the smallest plasma protein molecule (Table 1.2) is present in the largest concentration. Changes in albumin structure have the least effect of the protein on plasma viscosities, but the substance makes an important contribution to plasma viscosity through its high concentration in plasma. Many relationships have been suggested to express blood viscosity as a function of cell concentration, plasma viscosity and shear rate. When temperature is increased, the viscosity of blood and plasma is fallen. Measurements should be made at constant temperature (37°c).

Platelets and white cells, in general, have little influences on blood viscosity, because they are present

in very minute quantity as compared to red cells. The deformation of red blood cells allow the blood to remain fluid upto hematocrit of 98% ; rigid cell without deformation will cease to flow at a cell concentration of about 60%. The fall in viscosity with increasing shear rate is also a famifestation of the deformation of the erythrocytes (Murata [94]. The hematocrit also influence the deformation of the erythrocytes i.e. raising of the cell concentration produces an increase in cell deformation (Goldsmith [50] and therefore fall in viscosity of blood occures.

In stationary blood, rouleaux is formed which intracts to produce larger aggregates . At low flow rates the presence of these red cell structures strongly influences the viscosity of the blood. The size of the rouleaux and aggregates progressively decreases as the shear rate increases and this produces the typical shear thinning seen in blood at low shear rate. In normal blood the disaggregation is probably complete at shear rate 50 sec^{-1} approximately (Chien [33]). The viscosity of blood increases when the shear rate falls, but it is uncertain as to what happens when the shear rate actually falls to zero. The build up of a 3-d of increasing aggregates suggests that blood may show a yield stress.

1.3(b) Blood as a Non Newtonian Fluid :

Blood, from fluid mechanics point of view, can be considered as a neutrally buoyant suspension of erythrocytes

in a Newtonian liquid called plasma in small vessels the dimensions of red blood cells are no longer negligible as compared to the tube size, hence blood can not be considered as a homogeneous fluid in these tubes of diameter of less than 500μm. The flow of blood through this type of tube is of physiological and clinical importance. Due to its complex and anamalous behaviour it is very difficult to analyse it. The anamalies include Fahraeus-Lindqvist effect, Segre Silverberg effect, slip at the wall etc. This is specific nature of blood that exhibits a yield stress. Hence if applied shear stress is below a critical value, the response will be elastic and on removal of stress the shape of blood film is unaltered. Otherwise flow takes place and blood behaves as a non-Newtonian fluid. At higher values of shear rates the blood shows Newtonian characteristics with a limiting viscosity.

Some experiments by Bugliarello et al [16] and Cokelet [34] show that blood flow may deviate significantly from the Newtonian behaviour. Also it is found that suspended blood cells are responsible for the non-Newtonian nature of blood rheology through such mechanics as erythrocyte aggregation (cell protein interaction) and erythrocyte deformation. (cell-cell and cell plasma interaction) (Chien et al [31]). Anamalous behaviour of blood show that at low shear rates the blood exhibits yield stress and behaves like a Casson fluid. Charm and Kurland [25] modelled it is in terms of simple Power law fluid. The constitutive equation

suggested by Herschel and Bulkley has also been used to describe the shear rate dependence of blood. Charm and Karland [25] showed a limited shear rate change.

Iida and Murata [64] have studied the pulsatile flow of blood through narrow rigid circular tube using Herschel-Bulkley fluid model blood. Bugliarello and Sevilla [18] have assumed Casson fluid model of blood in the study and pulsatile blood flows in fine glass, tube. Arimah et al [3] have studied the similar problem through small rigid circular tube using micro-continuum approch and employing the condition of cell rotation at the solid boundary. Prahlad and Schultz [114] gave two fluid models for blood flow through small diameter tube.

Blood rheology can be helpful in diagonosis of some blood abnormalities (Merrill [9]) such as a decrease in sedimentation rate, increase in normal blood viscosity and yield stress would indicate Polythymia. Anaemia is usually indicated by lowering of yield stress and pronouncement of more Newtonian behaviour. Elevation of yield stress and extent of non-Newtonian regime beyond 200 Sec^{-1} would indicate Hyperfribrinogenimia.

1.4 SURVEY OF LITERATURE :

The German form of the word (Rheologie) and the description of a small viscometer as a "Microrheometer" are found in literature since early days; but, in its modern sense, the term Rheology was coined by Prof. E.C. Bingham

and formally adopted and defined at the foundation meeting of the American Society of Rheology in December, 1929 in Washington, D.C. as "The Science of the deformation and flow of matter".

However, the subject of hydrodynamics and aerodynamics are not included in rheology. Many authors (Lelievre et al [75] Nubber [100] etc.) have made studies on blood rheology.

Records show that the Greek and Romans were quite familiar with the anatomy of the heart and major blood vessels. Roman writings indicate that the valves of the heart were known to permit flow in only one direction. The knowledge concerning the mechanics of circulation remained at the level of the ancient Greeks and Romans until the renaissance when studies in anatomy led the way to the modern scientific approach.

The most remarkable observer Leonardo da Vinci (1452-1519) combined the study of structure with the the study of function through his detailed illustrations and notes on the cardio-vascular systems. Much of his thinking reflected the classical schemes, including the idea that the blood has an ebb and flow motion. He also remarked on the thickening and the hardening of the arterial walls, which we now call atherosclerosis. The modern concept of the circulation of blood was given by William Harvey in 1958.

The first mathematical paper of blood flow was given by Leonhard Euler in 1862. He developed the one dimensional equations for inviscid flow of an incompressible fluid in an elastic tube. His equations included both the conservation of mass and equations of motion. He also postulated a non-linear law relating the pressure at any point inside a blood vessel to its cross sectional area. Euler recognized that the heart could be thought of as a positive displacement pump and he gave a fairly complete statement of the governing equations of blood flow in the arteries. But he could not solved the problem posed by him. Recently, many workers have studied the flow property of blood (Bloor [9], Brunn [11], Copley [35], Goldsmith [56] Liepsch [77] Sugihara [ ] etc.

Blood flow in capillaries is of great interest to physiologists involved in micro-vascular research. The first direct observation of capillary vessels was reported by Malpight [85] in 1661. In the beginning of 19th century, the principal anatomical features such as flow velocities, pressure and viscous loss phenomena were known quantitatively as well. During 19th century, general knowledge of the circulatory system become much more detailed and quantitative.

The pressure drop relation in micro-circulation was obtained by Poiseuille in 1840. After this there appears to have been no further advance untill recent times in theoretical mechanics of viscous effects in

blood flow. The most remarkable discovery was of Fahraeus and Lindquist [44], showing that the apparent viscosity of blood decreases as tube diameter decreases from 500μm. This also has been confirmed by other investigators (Coklet [34], Dintenfass [39]). They observed experimentally that this effect is seen to occur with capillary radii of as little as 5μm, but this is reversed and the apparent blood viscosity increases with a further decrease in the capilarry radius. Recently, apparent viscosity and other properties were studied by Copley [35]. A variety of investigations were proposed by authors such as study of capillary wall, the elaboration of Fick's law of diffusion and its application to physiology etc. In biological science, the study of diffusivity of neutrients, metabolic products, drugs and other solutes is of at most importance. Specially, many life giving materials mixed in the blood reach different parts of the body by the process of diffusion. Taylor [151] studied dispersion process in Newtonian fluid flows through a circular tube. Patel and Sirs [110] and Rudraiah et al [121] made significant contributions in this direction.

1.5 Geometrical Aspect of Vessels :

A striking characteristic of the circulatory system is its geometrical complexity. Blood must flow through many bends, bifurcations, stenosis and tapering during its journey. While the complex anatomy of the major vessels has been known since ancient time, most of the

progress in understanding the mechanics of the circulation is based on the investigations of flow in straight uniform tubes. It is only within the last two decades that the effects of geometric transitions on blood flow have begun to be explained.

1.5(a) Stenosis :

A stenosis caused the narrowing of the blood vessles due to the development of abnormal tissues and gives way to serious circulatory disorder by reducing and occluding the blood supply. Hemodynamic characterestics may be changed this undersirable growth which could be injurious to normal health. In arterial system of mammals, it is quite common to find narrowings, some of which are at least, approximately, axisymmetric. These narrowing may be caused by intravescular plaque or the implingement of ligaments or squares on the vessel wall (Roach [119] Rodbard [120]

In recent years many workers have investigated the flow characterestic of blood through artery in the presence of mild stenosis (Abdallah and Hwang [1], Chakrawarty [21], Mahrotra and Jayraman [80] etc.).

Young [164] gave a theoretical analysis of the effect of time dependent stenosis on flow characterestics of blood. He concluded that resistance to flow (impedence) and the wall shear stress increase with increase in the stenosis size. Various authors have studied

Pulsatile effect of blood in mild stenosis. Due to presence of stenosis in the lumen, higher resistance to flow is caused and this effect becomes more prominant as the size of the stenosis increases, also shear stress increases with respect to stenosis height (Srivastava [137] and Young [164] .

The geometry of the constriction choosen was defined by a Guassian normal distribution curve. Several investigators have considered the numerical treatments of the symmetric constrictions in rigid two dimensional conduits. Mac Donald [91] , Morgan and Young [93] and Rand et al [117] considered the non-Newtonian behaviour in their studies. Recently, Sinha and Singh [130] studied the effect of stenosis on blood flow through the couple stress fluid model.

1.5(b) Tapering :

The mammalian arterial consists of the main aortic tube which continues into the two iliac and femoral arteries and a number of side branches. Transformations and propagation of pressure and flow waves depend on the distribution of the characterestic impedence. The two contributing factors are the decrease of cross sectional area and the peripheral increase of the wall stiffness (elastic tapering). Many blood vessels have the characterestic of taper cylinders rather than straight cylinders. This idea was given by Block [8]. He gave

the idea that the vessels which carry blood towards the tissues should be considerd as long, slowly tapering cones rather than cylinders. He obtained only pressure flow relation. The relations for shear stress and their variations with suspensiosn concentration and tapered angle, which are also important, have been discussed in this thesis.

TABLE 1.1
BASIC INFORMATION

	Av. diam.um	Rate of Shear at wall Sec^{-1}	Re. number
Large arteries	10,000	1,500	5,000
Terminal arteries	1,600	2,300	200
Arterioles	40	5,000	0.25
Capillaries	16	2,000	0.02
Venules	60	2,100	0.25
Main Veins	4,000	1,600	800
Large Veins	20,000	1,500	20

TABLE 1.2

PROTEIN CONCENTRATION IN HUMAN PLASMA

	Conc. (g/l)	Molecular Weight (ml)	Mol. Sizes (um)
Protein	75	-	-
Albumin	45	69,000	15 x 4
Globulin	27	1,50,000	24 x 4.5
Fibrinogen	3	3,40,000	70 x 4

CHAPTER II

Steady shear flow of micropolar fluid through a channel of Large Cross Section

Introduction :

The classical Navier - stobes theory does not adequately describe the flow properties of physiological fluids, e.g. blood, synovial fluid, etc. Mathematical model for the description of such fluids, which exihibit certain microscopic effects arising from the local structure and micromotion of the fluid element, is categorically introduced by A.C. Eringen (1964, 66). A subclass of these fluids is the micropolar fluids which have the microrotational effects and microrotational inertia. Ariman et al (1974) reported that the theoretical results of polar fluid have good agreement with the experimental data of steady and pulsatile blood flow given by Bugliarello and sevilla (1970).

In the present study we have proposed to analyse the fully developed steady shear flow of micropolar fluid betweet two parallel plates. The polarity arising due to the presence of buoyant corpuscles in the fluid leads to give a theoretical insight of the blood flow through large arteries. The microrotational field depends on the length ratio parameter. Microrotational effects and microrotational inertia are considered in this approach. Exact steady solution for velocity and microrotational subject to the strong and weak limit of influence of surface on microrotation are obtained. The parameters characterizing the ratio of vortex viscosity to shear viscosity and length ratio which do not appear in a classical Newtonian fluid are introduced and its effects on velocity and microrotation are discussed.

Mathematical analysis :

Consider the constitutive equations for micropolar fluid in the form

$$\tau_{ij} = (-p + \mu' d_{kk})\,\delta_{ij} + 2\mu\, d_{ij} + 2\,\mu_1\,\varepsilon_{ijk}\,(W_k - \sigma_k) \qquad (1)$$

$$M_{ij} = \alpha\,\sigma_{p,p}\,\delta_{ij} + \beta\,\sigma_{i,j} + \gamma\,\sigma_{j,i} \qquad (2)$$

Where τ_{ij} is stress tensor

M_{ij} is couple tensor

ε_{ijk} is alternating tensor

σ_k is microrotation

W_k is vorticity

p is thermodynamic pressure

$d_{ij} = \frac{1}{2}(V_{i,j} + V_{j,i})$

$W_k = \varepsilon_{kij}\, V_{j,i}$

μ and μ' are the viscosity coefficient of classical fluids and μ_1, α, β, γ are viscosity coefficients of micropolar fluids. These coefficient are subject to the restruction

$$\mu_1 > 0,\ \mu > 0,\ \gamma \geq 0$$

$$3\mu' + 2\mu > 0,\ 3\alpha + \beta + \gamma \geq 0,\ -\gamma \leq \beta \leq \gamma$$

Incompressible flow of micropolar fluid between two fixed parallel infinite plates at distance h apart is considered. The flow throughout the region is maintained by a constant pressure gradient $\frac{dp}{dz}$. The lower plate with origin is assumed to occupy the plane y=0 and the upper plate is at y=h. At sufficiently large distance from the entrance region, the flow is supported to be fully developed. In the steady state the velocity $\bar{v}$ and microrotation $\bar{\sigma}$ at any point have the component $(0, 0, \bar{u})$ and $(0, 0, \bar{\Omega})$, respectively. We have further supposed that the orientation of vorticity and microrotation are parallel. Under these assumption and in view of equations (1) and (2). The equations of motion can be written as

$$(\mu + \mu_1)\frac{d^2\bar{u}}{dy^2} + 2\mu_1\frac{d\bar{\Omega}}{dy} - \frac{dp}{dx} = 0 \qquad (3)$$

$$\gamma \frac{d^2 \bar{\Omega}}{dy^2} - 2\mu \frac{d\bar{u}}{dy} - 4\mu_1 \Omega = 0 \qquad (4)$$

On introducing the following non dimensional quantities

$$\eta = \frac{y}{h}, \quad u = \frac{\bar{u}}{(Ah^2/2\mu)}, \quad \Omega = \frac{\bar{\Omega}}{(A\ h/2\mu)}$$

$$\alpha = \frac{\mu_1}{\mu}, \quad \lambda = \frac{h}{[\gamma\ (\mu + \mu_1)/\ 4\mu\ \mu_1]^{1/2}}$$

the equations (3) and (4) reduced to

$$-1 + \tfrac{1}{2}\ (1+\alpha)\ u'' + \alpha\ \Omega' = 0 \qquad (5)$$

$$\Omega''' - \lambda\ \Omega' - \lambda = 0 \qquad (6)$$

where premes denote differentiation with respect to η.

The equations (5) and (6) are solved under two limiting boundary conditions.

i) No spin boundary which corresponds to no-slip condition, $\Omega(o) = \Omega(1) = 0$. This is based on the argument that fluid-solid interface with interaction is so strong that the microstructure does not rotate relative to the plate.

ii) Weak limit of influence of microrotation on the surface given by $\Omega'(0) = \Omega'(1) = 0$. This is based on the fact that the corpuscles rotatate on the wall.

In view of these two conditions and no-slip velocity at boundary, $u(0) = u(1) = 0$, the solutions are.

$$\Omega s = \text{cosec h}\ \lambda\ \sinh \lambda\ \eta - \eta \qquad (7)$$

$$us = \frac{2\alpha\ \text{cosec h}\ \lambda\ [1-\eta+\eta\ \cos h\lambda - \cos h\lambda\eta] - \eta\ (1-\eta)}{\lambda(1+a)} \qquad (8)$$

$$\Omega_w = \frac{\text{cosec h}\ \lambda\ [\cos h\ \lambda\ \eta - \cos h\ (1-\eta)\ \lambda]}{\lambda} - \eta \qquad (9)$$

$$Uw = \frac{2\alpha\ [\eta+(1-\eta)\cot h\ \lambda + \{\eta - \sin h\ \lambda\eta - \cos h(1-\eta)\lambda\}\text{cosec h}\ \lambda\}] - \eta(1-\eta)}{\lambda^2\ (1+\alpha)} \qquad (10)$$

subscripts s and w denote the values at strong and weak limits of interaction of fluid-solid interface, respectively.

Discussion :

The numerical values of microrotation and velocity for different field parameters are obtained and their variations in the flow regions are encountered,From the table we observe that all the values obtained are negative. This shows that backward flows occur which is quite obvious because the pressure gradient under whose influence the flow is assumed to be maintained, increases along x axis and thus the flow tabes place from the high pressure to low pressure direction. From table I we infer that having all its physical properties to the constant, when the width of the channel increases then the microrotation Ω increases for strong limit of interaction while for the weak limit of interaction an opposite behaviour is seen. Below the central region. From table II we see that for small α velocity field increases with λ below the central region and above it a reverse trend follows, whereas for weak limit of interaction a smooth increase in velocity with λ is recorded throughout the whole region. Also, when α increases velocity field decreases.

TABLE 9

Variation of microrotational field

η	Ωs			Ωw		
	$\lambda=1$	$\lambda=3$	$\lambda=5$	$\lambda=1$	$\lambda=3$	$\lambda=5$
0.0	-0.0000	-0.0000	-0.0000	-0.4621	-0.3017	-0.1073
0.1	-0.0147	-0.0696	-0.0929	-0.4642	-0.3138	-0.2182
0.3	-0.0408	-0.1975	-0.2713	-0.4786	-0.3908	-0.3383
0.5	-0.0565	-0.2874	-0.4184	-0.5000	-0.5000	-0.5000
0.8	-0.0442	-0.2543	-0.4322	-0.5300	-0.6545	-0.7305
0.9	-0.0265	-0.1607	-0.2935	-0.5357	-0.6861	-0.7817
1.0	+0.0000	-0.0000	-0.0000	-0.5378	-0.6982	-0.8026

TABLE 99

Variation of velocity field (α =0.2)

η	Us			Uw		
	λ=1	λ=3	λ=5	λ=1	λ=3	λ=5
0.0000	-0.0000	-0.0000	-0.0000	-0.0000	-0.0000	-0.0000
0.1	-0.0760	-0.0804	-0.0835	-0.0692	-0.0811	-0.0847
0.3	-0.1766	-0.1846	-0.1916	-0.1609	-0.1908	-0.2006
0.5	-0.2091	-0.2147	-0.2219	-0.1903	-0.2294	-0.2387
0.8	-0.1324	-0.1300	-0.1312	-0.1201	-0.1445	-0.1516
0.9	-0.0742	-0.0712	-0.0705	-0.671	-0.0808	-0.0847
1.0	-0.0000	-0.0000	-0.0000	-0.0000	-0.0000	-0.0000

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2. Eringen, A.C., J. Math. Mech. 16, 1, (1966).
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Chapter III

Steady flow of blood in narrow tube through micropolar fluid model with slip at the wall.

Introduction :

Blood, a class of fluid has its component as plasma, platelets, erythrocytes and leubocytes. But most of its rheological properties depend on RBC which in normal condition occupy some 40-45% fraction of total blood volume. The general evidence is that the flow processes do not obey the same rule everywhere as they vary from small to large diameter tube. In absence of our knowledge of cellular geometry and the flow inside the cell we assume it a rigid particle of finite dimension suspended in base fluid plasma. In narrow tubes (20-500μm; 1 μ m = 10^{-6} m) we may propose the micropolar fluid theory as a model to explain the flow parameters characterizing the blood behaviour. Several authors like Klime (1968), Brunn (1975), Ariman (1974), Cowin (1972) Erdogen (1980) and Chaturani (1984) have proposed micropolar fluid theory a suitable candidate for the same. They have used different boundary conditions at the wall.

Further, it is found to be more realistic to consider the shear viscosity of blood as different from that of solvent viscosity. In the present section we have studied the poiseuille flow of micropolar fluid as a model of blood flow in 40 μm diameter tube and for 40% RBC concentration.

At the boundary we have introduced a slip in the axial velocity and a partial rotation depending upon the wall effect parameter S and fluid vorticity. From the analysis we have observed the the introduction of slip reduces the apparent viscosity of the suspension and increases its axial velocity whereas it has no effect on particle rotation. Effect of $\bar{S}$ ($0 \leq \bar{S} \leq 1$, a parameter depending on S and concentration) on different flow parameters are seen and

by comparing the results with the experimental value we have tried to estimate the value of $\bar{s}$.

Mathematical Analysis :

Fully developed steady incompressible flow of micropolar fluid through a rigid circular tube of radius R has been considered. As a model for the flow of blood in narrow vessels, neglecting body forces and body couples, we write the equation of motion in the form :

$$(\mu + \tau)\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v}{\partial r}\right) + 2\tau\frac{1}{r}\frac{\partial}{\partial r}(rw) - \frac{dp}{dz} = 0 \qquad (1)$$

$$\gamma\frac{\partial}{\partial r}\left[\frac{1}{r}\frac{\partial}{\partial r}(rw)\right] - 2\tau\frac{\partial v}{\partial r} - 4\tau w = 0 \qquad (2)$$

where [0, 0, v(r)] is the velosity, w(r) the total angular velocity; r, z the radial and axial co-ordinates, p the pressure, μ the shear viscosity, τ the rotational viscosity and γ the rotation parameter.

Boundary conditions for the flow are

at r = 0; v and w remain finite

$$\text{at } r = R;\ v = -\sigma\frac{\partial v}{\partial r},\ w = -\frac{s}{2}\frac{\partial v}{\partial r}\ ;\ (0 \le \bar{s} \le 1) \qquad (3)$$

where σ - the slip parameter, s - the wall parameter.

Solution of equations (1) and (2) in view of (3) are

$$v = \frac{K R^2}{4\mu}\left[1-y^2 - 2\eta\bar{s}\left\{\frac{I_0(\lambda) - I_0(\lambda y)}{\lambda I_1(\lambda)}\right\}\right] + 2\chi(1-\eta\bar{s}) \qquad (4)$$

$$w = \frac{K R}{4\mu}\left[y - \frac{\bar{s} I_1(\lambda y)}{I_1(\lambda)}\right] \qquad (5)$$

Where = $y = r/R$, $\eta = \sigma/R$, $K = -\frac{dp}{dz}$, $\lambda^2 = \frac{4R^2\mu\tau}{\Gamma(\mu+\tau)}$

$$\eta = \frac{\tau}{\mu + \tau}, \quad \bar{s} = \frac{1 - s}{1 - \eta s} \quad ; \quad (0 \leq S \leq 1)$$

I_n is the modified Bessel function of order n.

From equation (4), the flow rate Q, apparent viscosity μ_a and velocity Va at tube axis are obtained as

$$Q = \frac{\pi R^4 K}{8 \mu} \left[1 - 4\eta \bar{s} \left\{ \frac{I_0(\lambda)}{\lambda I_1(\lambda)} - \frac{2}{\lambda^2} \right\} + 4 \psi (1 - \eta \bar{s}) \right] \quad (6)$$

$$\mu_a = \mu \left[1 - 4\eta \bar{s} \left(\frac{I_0(\lambda)}{\lambda I_1(\lambda)} - \frac{2}{\lambda^2} \right) + 4 \psi (1 - \eta \bar{s}) \right]^{-1} \quad (7)$$

$$V_a = \frac{KR^2}{4\mu} \left[1 - 2\eta \bar{s} \left(\frac{I_0(\lambda) - 1}{\lambda I_1(\lambda)} \right) + 2 \psi (1 - \eta \bar{s}) \right] \quad (8)$$

Discussion :

From equation (4) and (5) we obsere that slip at the boundry affeels the axial velocity whercas the particle rotatoin is unaffeted.

The boundry condition parameter B_t of Brumn (1975) analysis is related with S by the relation $B_t = \left(\frac{1}{S} - 1\right) \frac{\tau}{\mu}$; $B_t \geq 0$ for the discussion we have considered two case. In case (I) we suppose $\mu = \mu_s + \tau$ and in case II we suppose $\mu = \mu_s$ where μ_s is the solvent viscosity. We see that the results obtained for case I are in conformity with the experimental value, wheras the results of case II are in parity to the results of Erdogen (1980) analysis which on comparing with the experimental value give unsatifactory reading for 40% concentration and 40 μm tube we have used.

$\mu_s = 1.2\ c_p$, $\tau = 0.98\ c_p$, $\Gamma = 12 \times 10^{-8}$ gm cm/sec. from Ariman (1974) analysis.

For exerimental verification we have used the data of Bugliarello and Sevilla (1970).

For different values s, ψ, the theretical values of μ_a, v_a, $-\frac{dp}{dz}$, v(r) and w(r) are shown in tables I - V.

For ψ = 0.02, the apparent viscosity μ_a varies from 2.02 to 2.8 where as for ψ = 0 this range is from 2.18 to 3.109 which is greater than the pervious one. From the analysis of Bugliarello and Sevilla, the experimantal value of μ_a for 40% concentration could be read any where between 2.1 and 2.5. To discuss the other parameters we have chosen the value of μ_a arbitrarily equal to 2.5 c_p. We are not known with any precise experinental value of $\bar{s}$. It can be estimated only from equation (7) as in it other quantities except $\bar{s}$ are experimentally known. The values of $\bar{s}$ lying between 0.35 and 0.66 give better theoritical result. We can approximate $\bar{s}$ = 0.5 because for this.

Value of $\bar{s}$ there are small errors in μ_a and V_a when compared with experimantal values.

Variation of pressure gradient and axial velocity with respect to ψ and $\bar{s}$ are reported in table II.

Variations of velocity and rotational. Profiles for different $\bar{s}$ are shown in tables III, IV. From tables we observe that when $\bar{s}$ increases, particles axial and rotational veloities decrease. If $\bar{s}$ is assumed to be the measure of concentration, then it is seen that for dilute suspension, particles rotation increases with tube radius (a case of Newtonian fluid) . But for sufficently large value of $\bar{s}$ ($\bar{s}$ > 0.85) the rotation at first increases, attains its maximum value and then decreases near the wall. For maximum concetration ($\bar{s}$ = 1) the particle rotaion is maximum of r = 0.75 which has been experimentally observed.

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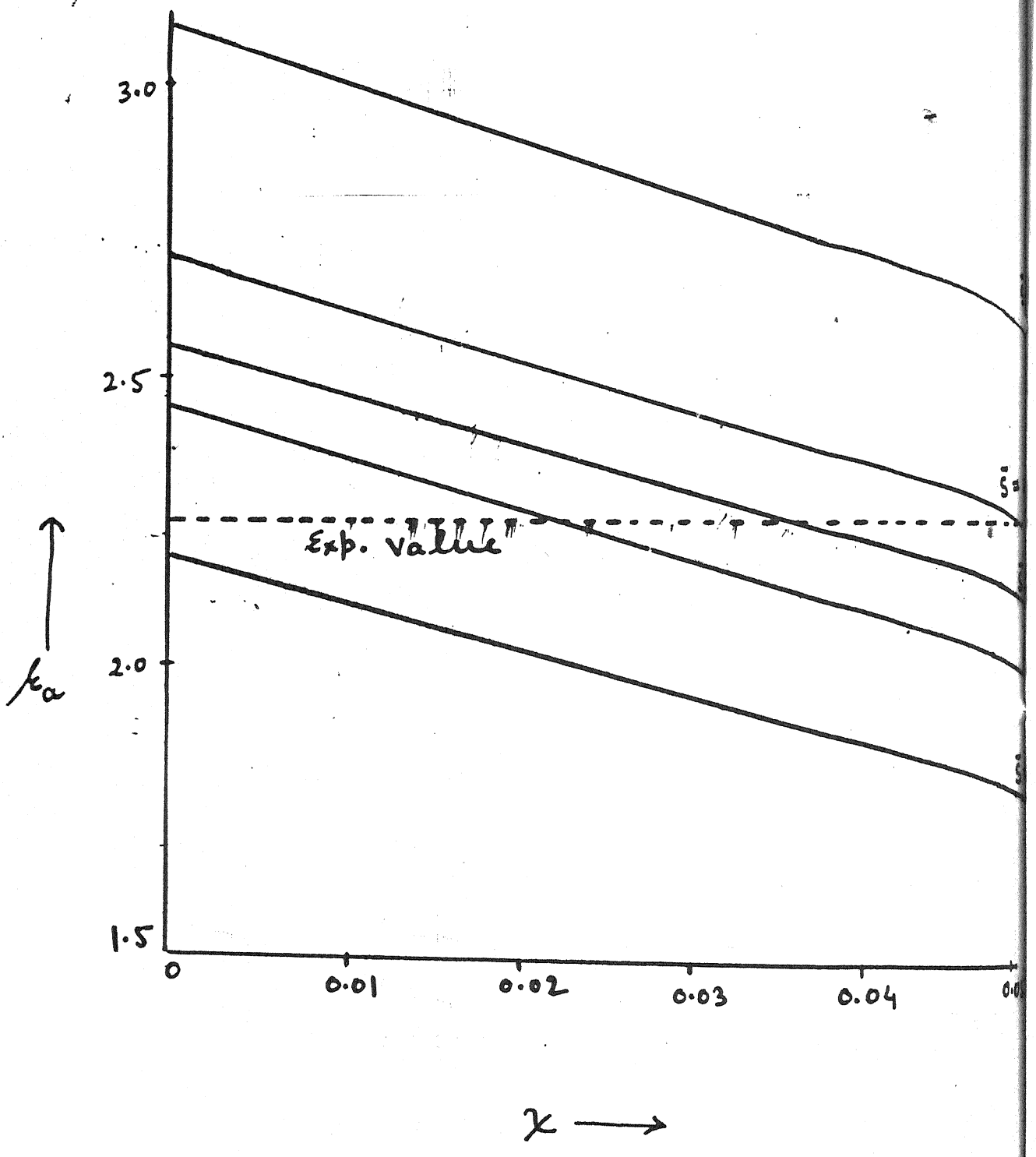
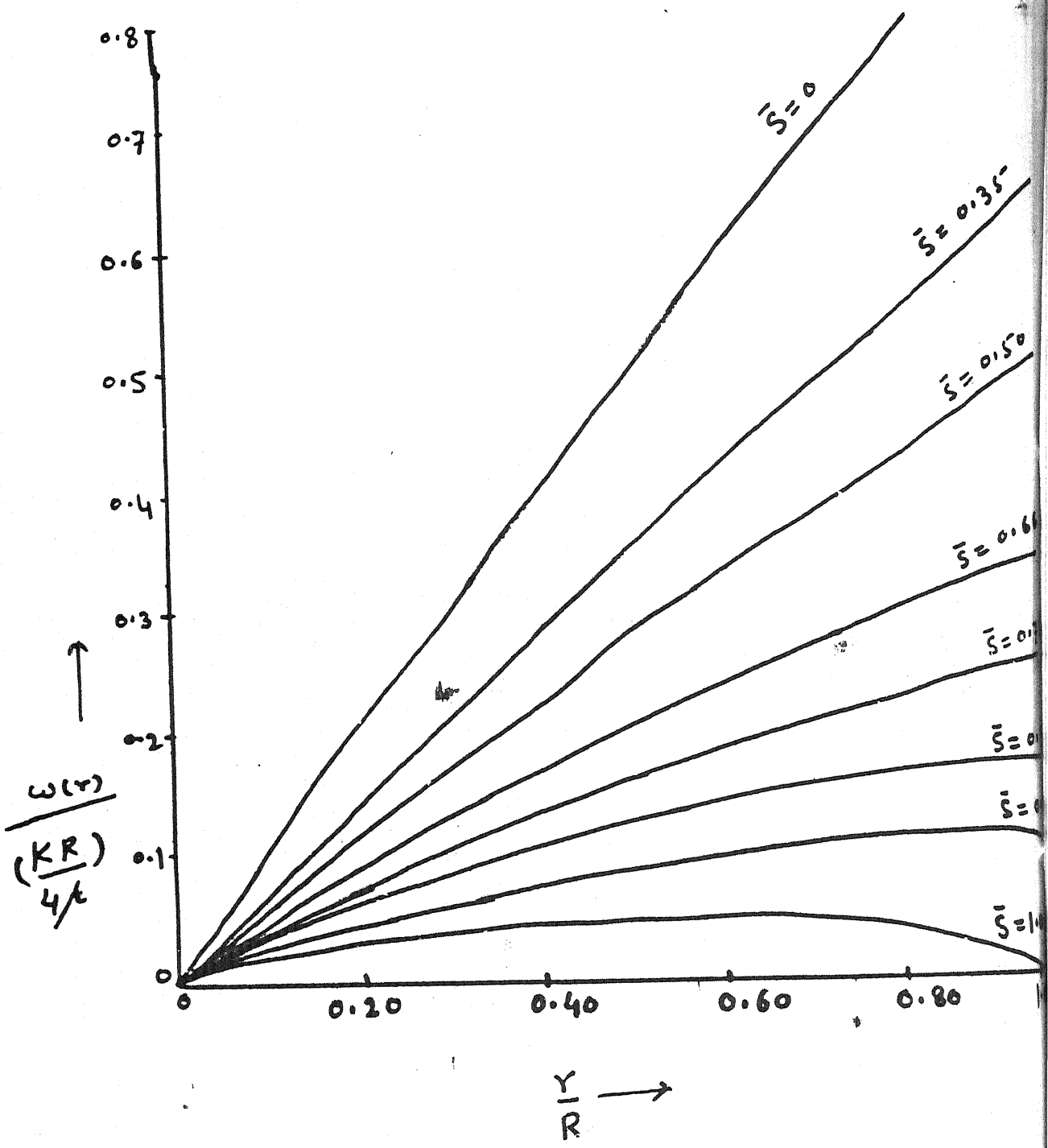


Fig.(4). Variation of apparent viscosity μ_a.



Fig(2). Variation of rotational velocity with $\bar{S}$

of a polar fluid under various boundry conditions.with application to blood flow. acte. 23, 435, 1984.

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Table I

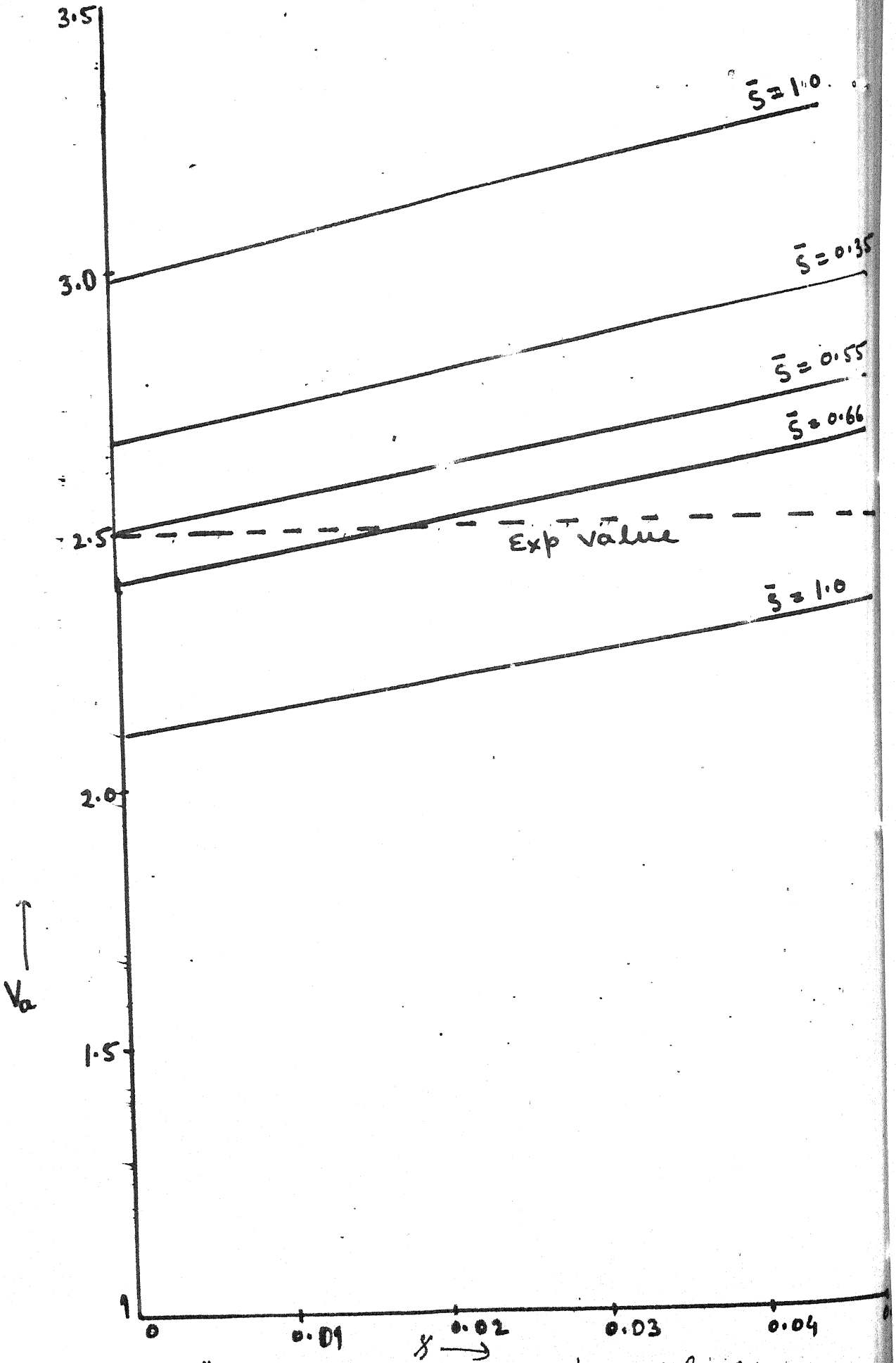
Variation of μ_a, V_a and V_s for different values of $\bar{s}$

ψ	$\bar{s}$	μ_a	V_a	V_s (Slip velocity)
0.2	0.000	2.02	3.11	0.119
0.02	0.265	2.19	2.85	0.109
0.02	0.350	2.25	2.79	0.106
0.02	0.491	2.36	2.66	0.101
0.02	0.520	2.39	2.63	0.100
0.02	0.666	2.52	2.50	0.095
0.02	1.000	2.88	2.20	0.082

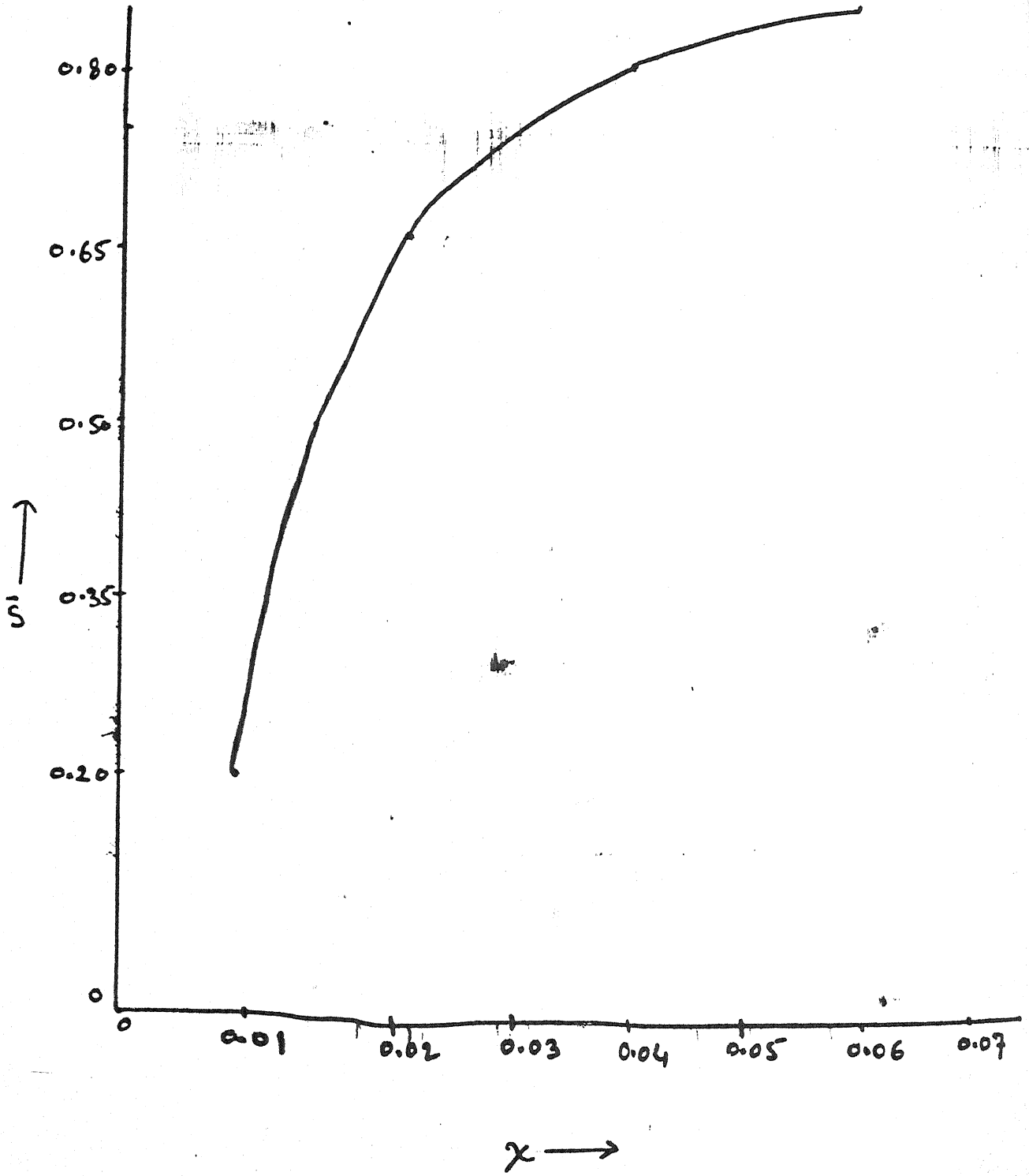
Table II

Variation of $-\frac{dp}{dz}$ V_a and V_s with respect to ψ and $\bar{s}$

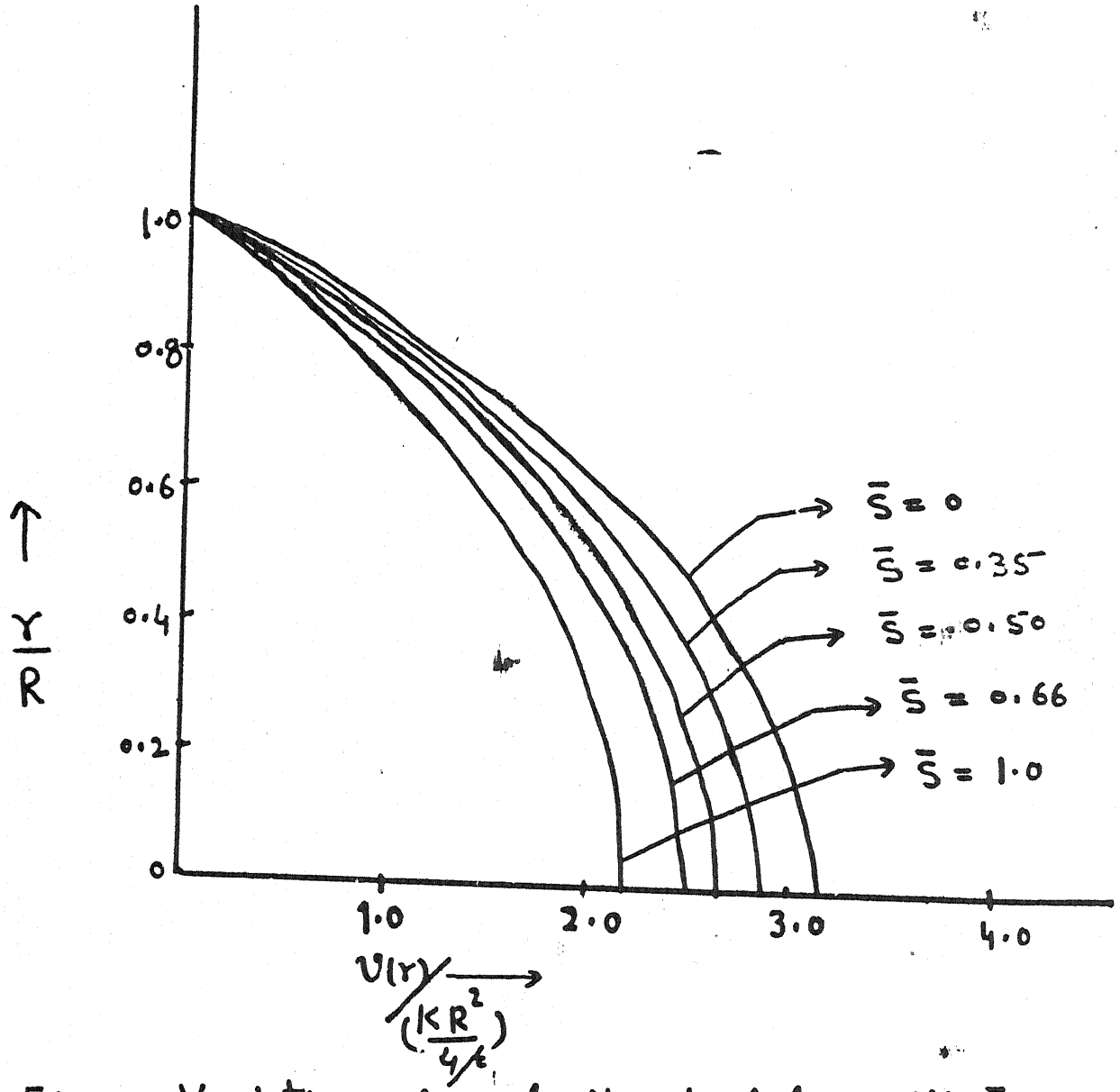
ψ	$\bar{s}$	-dp/dz	Va	Vs
0.01	0.591	65.31	2.47	0.048
0.02	0.591	64.27	2.53	0.097



Fig(2) Variation of Maximum axial Velocity



Fig(5). Variation of x with $\bar{s}$.



Fig(1): Variation of velocity profiles with $\bar{S}$,

Variat
for

$\bar{S}$
0.000
0.350
0.500
0.666
1.000

Table V

ariation of appar ent viscosity and axial velocity for different values of $\acute{S}$ at constant $\psi = 0.02$

	μ_a	V_a	V_s
00	1.11	5.66	0.217
50	1.31	5.35	0.183
00	1.42	4.44	0.152
66	1.56	4.03	0.152
00	1.97	3.22	0.119

CHAPTER IV

Study of dispersion processes in blood flow in narrow vessels

Introduction :

In biological sciences, the study of diffusivity of neutrients, metabolic products, drugs and other solutes is of utmost importance. Specially, many life giving materials mixed in the blood reach to the different parts of the body by the process of diffusion. Taylor (1953, 54) studied the dispersion process in Newtonian flow and discussed the effective dispersion coefficient with respect to the average speed of the flow, the radius of the tube and molecular diffusion coefficient. Many authors as Patel and Sirs (1983), Federspial and Fredberg (1988), Rudraiah et. al (1986) have studied dispersion processes by taking different flow models. Shukla and Gupta (1981) studied the Taylor dispersion in Bingham Plastic fluid surrounded by Newtonian peripheral layer. Scottblair and spanner (1974), Charm and Kurland (1968), Bugliarello and Sevilla (1970) and many others have proposed that casson model has an edge over the other fluid models for the purpose of discussing blood behaviour.

In our problem we have proposed the flow model with casson fluid in the core region surrounded by a Newtonian plasma layer near the wall. Taylor's limiting condition and Ficks law of diffusion are used for finding out the solution of the problem. The effective dispersion coefficient with which the solute disperses across a plane moving with mean speed of the medium is found to be decreased with respect to the yield stress and molecular diffusion coefficient whereas a reciprocal behaviour is observed with respect to the viscosity of the casson fluid.

Mathematical analysis :

Consider an incompressible steady viscous flow of casson fluid through a circular tube of radius R. During motion, the viscosity and yield stress of fluid vary along the radial direction. The constitutive equation in one dimensional form of casson fluid is :

$$\tau^{1/2} = \tau_y^{1/2} K^{1/2}(r) + \eta^{1/2}(r) \dot{\gamma}^{1/2}; \quad \tau > \tau y\, K(r)$$

$$\gamma = 0; \quad \tau \leq \tau_y K(r) \qquad (1)$$

where $\eta(r)$ is the viscosity, $\tau_y K(r)$ is the yield stress. γ is the strain rate. $\eta(r)$ and $K(r)$ are assumed to be decreasing function of r. The region $0 \leq r \leq R$ is divided into a central region $0 \leq r \leq R_1$ of casson fluid with a plug radius R_0, $0 \leq r \leq R_0 \leq R_1$ and by a peripheral region of another casson fluid ($R_1 \leq r \leq R$). The function $\eta(r)$ and $K(r)$ are assumed to be

$$\eta(r) = \eta_1; \; K(r) = 1 \text{ when } 0 \leq r \leq R_1$$

$$\eta(r) = \eta_2; \; 0 \leq K(r) = K < 1; \text{ when } R_1 \leq r \leq R \qquad (2)$$

Where η_1 and η_2 are the viscosities of the central and the peripheral fluids. In case of blood, the peripheral layer is a Newtonian fluid and thus the function $K(r) = k = 0$ in $R_1 \leq r \leq R$.

For one dimensional steady laminar flow in cylindrical co-ordinate system (r, x, θ) whose origin lies on the axis of the vessel, the equation of motion can be written as

$$\frac{-dp}{dx} = \frac{1}{r} \frac{d}{dr} (r\tau) = 0 \qquad (3)$$

Integrating equation (3) and using the boundary condition

$\tau = 0$ at $r=0$, we obtain,

$$\tau = \frac{-dp}{dx} \frac{r}{2} \qquad (4)$$

For plug boundary equation (4) gives

$$\tau_y K(r) = \frac{-dp}{dx} \frac{R_0}{2} \qquad (5)$$

In view of equations (4), (5) and boundary condition $V_x=0$ at r=R, the integration of equation (1) gives

$$V_x = \left(\frac{-R^2}{2\eta_1}\frac{dp}{dx}\right)\left[\int_y^1 \frac{y\,dy}{\bar{\eta}(y)} + \frac{y_0}{K(y0)}\int_y^1 \frac{K(y)\,dy}{\eta(y)} - \frac{2y_0^{1/2}}{K^{1/2}(y_0)}\int_y^1 \frac{K^{1/2}(y)\,y^{1/2}\,dy}{\eta(y)}\right]$$

$$; y_0 \le y \le 1 \qquad (6)$$

Where $y = \frac{r}{R}$; $y_0 = \frac{R_0}{R}$, $\bar{\eta}(y) = \frac{\eta(y)}{\eta_1}$ (7)

For $y = y_0$, the plug velocity V_x is calculated from equation (6) as

$$V_{x_0} = \left(\frac{-R^2}{2\eta_1}\frac{dp}{dx}\right)\left[\int_{y_0}^1 \frac{ydy}{\bar{\eta}(y)} \frac{y_0}{K(y_0)}\int_{y_0}^1 \frac{K(y)dy}{\bar{\eta}(y)} - \frac{2y_0^{1/2}}{K^{1/2}(y_0)}\int_{y_0}^1 \frac{K^{1/2}(y)y^{1/2}}{\bar{\eta}(y)}\,dy;\right]$$

$$0 \le y \le y_0 \qquad (8)$$

The average velocity $\bar{V}$, of the fluid is obtained as

$$\bar{V} = \frac{1}{\pi R^2}\int_0^R 2\pi r\,Vx\,dr = -\int_0^1 y^2\frac{dVx}{dy}\,dy$$

Using equations (6) and (8) in above equation, we get

$$\bar{V} = \left(\frac{R^2}{2\eta_1}\frac{dp}{dx}\right)\left[\int_{y_0}^1 \frac{y^3dy}{\eta(y)} + \frac{y_0}{K(y_0)}\int_{y_0}^1 \frac{y^2\,K(y)}{\eta(y)}\,dy \frac{2y_0^{1/2}}{K^{1/2}(y_0)}\int_{y_0}^1 \frac{y^{5/2}\,K^{1/2}(y)}{\bar{\eta}(y)}\,dy\right] \qquad (9)$$

Now, the equation for concentration is

$$\frac{\partial c}{\partial t} + V_x\frac{\partial c}{\partial x} = \frac{1}{r}\frac{\partial}{\partial r}\left[r\,D(r)\frac{\partial c}{\partial r}\right] \qquad (10)$$

Where D(r) is moleculer diffusion coefficient assumed to vary symmetrically along r- direction.

Now introducing the non-dimensional quantities

$$\theta = \frac{t}{\bar{t}},\ \bar{t} = \frac{L}{\bar{V}},\ \xi = \frac{x - \bar{V}\,t}{L},\ y = \frac{r}{R},\ \bar{D} = \frac{D}{D_1}$$

Where L and D are the typical values of The length along the tube and molecular diffusion coefficient, respectively. Equation (10) is transformed to give flow of solute relative to a plane moving with the mean speed of the flow as

$$\frac{1}{\bar{t}}\frac{\partial c}{\partial \theta} + \frac{(Vx - \bar{V})}{L}\frac{\partial c}{\partial \xi} = \frac{D1}{R^2 y}\frac{\partial}{\partial y}\left(y\,\bar{D}\,\frac{\partial c}{\partial y}\right) \qquad (11)$$

Applying Taylors limiting conditions ($\frac{\partial c}{\partial \xi}$ is independent of y and $\frac{\partial c}{\partial \xi}=0$), equation (11) gives

$$\frac{1}{y}\frac{\partial}{\partial y}\left(\bar{D}\,y\,\frac{\partial c}{\partial y}\right) = F\,g\,(y) \qquad (12)$$

Where

$$g(y) = \int_{y_0}^{1}\frac{y\,dy}{\bar{\eta}(y)} + \frac{y_0}{K(y_0)}\int_{y_0}^{1}\frac{K(y)\,dy}{\bar{\eta}(y)} - \frac{2y_0^{1/2}}{K^{1/2}(y_0)}\int_{y_0}^{1}\frac{y^{1/2}\,K^{1/2}(y)}{\eta(y)}\,dy - G;$$

$$; \; 0 \le y \le y_0$$

$$g(y) = \int_{y_0}^{1}\frac{y\,dy}{\eta(y)} + \frac{y_0}{K(y_0)}\int_{y_0}^{1}\frac{K(y)\,dy}{\eta(y)} - \frac{2y_0^{1/2}}{K^{1/2}(y_0)}\int_{y_0}^{1}\frac{y^{1/2}\,K^{1/2}(y)}{\eta(y)}\,dy - G;$$

$$; \; y_0 \le y \le 1$$

$$G = \int_{y_0}^{1}\frac{y^3 dy}{\bar{\eta}(y)} + \frac{y_0}{K(y_0)}\int_{y_0}^{1}\frac{y^2 K(y)\,dy}{\bar{\eta}(y)} - \frac{2y_0^{1/2}}{K^{1/2}(y_0)}\int_{y_0}^{1}\frac{y^{5/2}\,K^{1/2}(y)\,dy}{\bar{\eta}(y)} \qquad (14)$$

$$F = \left(-\frac{1}{2\eta_1}\frac{dp}{d\xi}\right)\frac{R^4}{D_1 L^2}\frac{\partial c}{\partial \xi} \qquad (15)$$

Integrating equation (12) and using conditions c=0 at $\beta=0$; $\frac{\partial c}{\partial \beta}=0$ at $\beta=1$, we obtain

$$C = C_0 - F\,M\,(y) \qquad (16)$$

$$\text{Where } M(y) = \int_0^{y}\frac{1}{\bar{D}y}\left\{\int_y^{1} y\,g(y)\,dy\right\}dy \qquad (17)$$

The average volumetric flow rate $\bar{Q}$ is given by

$$\bar{Q} = 2\int_0^{1} y\,(V_x - \bar{V})\,c\,dy$$

Using equations (6), (8), (9) and (16) in the above equation we get

$$\bar{Q} = -\frac{2\bar{V}^2 R^2}{D_1 L}\left[\int_{y_0}^{1}\frac{y^3\,dy}{\bar{\eta}(y)} + \frac{y_0}{K(y_0)}\int_{y_0}^{1}\frac{y^2 K(y)}{\bar{\eta}(y)}\,dy - \frac{2\,y_0^{1/2}}{K^{1/2}(y_0)}\int_{y_0}^{1}\frac{y^{5/2}\,K^{1/2}(y)\,dy}{\bar{\eta}(y)}\right]^2 \int_0^{1} y\,M(y)\,g(y)\,dy$$

(18)

Compare equation (18) with the Ficks law of diffusion

$$J^* = -D^* \frac{\partial c}{\partial x}$$

The solute disperses relative to a plane moving with the menan speed of flow with an effective dispersion coefficient D*, given by

$$D^* = \frac{2R^2\bar{V}}{D_1} \frac{\int_0^1 y\, M(y)\, g(y)\, dy}{\left[\int_{y_0}^1 \frac{y^3 dy}{\bar{\eta}(y)} + \frac{y_0}{K(y_0)} \int_{y_0}^1 \frac{y^2 K(y)}{\bar{\eta}(y)} dy - \frac{2y_0^{1/2}}{K^{1/2}(y_0)} \int_{y_0}^1 \frac{y^{5/2} K^{1/2}(y)}{\bar{\eta}(y)} dy\right]^2} \quad (19)$$

The expression for D* is applicable for any general functions $\bar{\eta}(y)$, $\bar{D}(y)$ and K(y).

Now, for the effect of a peripheral layer around a casson fluid, the boundary condition for the functions $\bar{\eta}(y)$, $\bar{D}(y)$ and K(y) may be assuemed as follows :

$$\bar{\eta}(y) = \begin{cases} 1 & ; 0 \le y \le y_1 = \frac{R_1}{R} \\ \frac{\eta_2}{\eta_1} = \mu & ; y_1 \le y \le 1 \end{cases} \quad (20)$$

$$\bar{D}(y) = \begin{cases} 1 & ; 0 \le y \le y_1 \\ \frac{D_2}{D_1} = \nu & ; y_1 \le y \le 1 \end{cases} \quad (21)$$

$$K(y) = \begin{cases} 1 & ; 0 \le y \le y_1 \\ 0 & ; y_1 \le y \le 1 \end{cases} \quad (22)$$

Where y1, η_1 and D_1 are the radius, viscosity and ,molecular diffusion coefficient of casson fluid respectively and η_2, D_2 are peripheral layer fluid.

Using equations (20), (21) and (22) in equations (13), (14), (17) and (19) we obtain

$$D^* = \frac{R^2 \bar{V}^2}{D_1} H \quad (23)$$

Where

$$H = \frac{S_0 \left(S_1 - S_2/\nu\right)}{\left[\, 1-(1-\mu)\, y_1^4 - \frac{y_0^{1/2}\mu}{21}\left\{y_0^{7/2}\mu - 4y_1^3\,(7y_0^{1/2} - 12\, y_1^{1/2})\right\}\right]}$$

$$S_0 = 1 - (1-\mu)\,(2-y_1^2)\,y_1^2 - \frac{y_0^2\mu}{21}\,(14-y_0^2) + \frac{4y_0y_1\mu}{3}\,(3-y_1^2)$$

$$- \frac{16y_0^{1/2}\, y_1^{3/2}\mu}{21} \times (7-3y_1^2)$$

$$S_1 = \frac{y_1^2}{2}\log_e y_1 - \frac{y_1^2}{4} - \frac{y_1^4}{8},$$

$$S_2 = S_1 + \frac{3}{8},$$

Discussion :

For fixed mean speed of the flow, the effects of μ, y_0 and ν, namely the viscosity, yield stress and molecular diffusion coefficient on H are seen from the tables (I), (II) and (III). We find that the magnitude of H decreases as the yield stress and molecular diffusion coefficient increases but increases with respect to the coefficient of viscosity.

Table I

Effects on μ, ν and y_0 on H, (ν = 1.1, y_1 = 0.90)

	H			
y_0	μ =0.20	0.40	0.60	0.80
0.30	-0.0480	0.0873	-01316	-0.1710
0.05	-0.0321	0.0680	-0.1034	-0.1360
0.09	-0.0219	-0.0450	-0.0689	-0.0928

Table III

(ν =1.3, y_1=0.90)

	H			
y_0	μ=0.20	0.40	0.60	0.80
0.03	-0.0401	-0.0857	-0.1292	-0.1679
0.05	-0.0315	-0.0668	-0.1015	-0.1336
0.09	-0.0215	-0.0442	-0.0677	-0.0904

Table III

ν = 1.5, y_1 = 0.90

	H			
y_0	μ = 0.20	0.40	0.60	0.80
0.03	-0.0395	-0.0846	-0.1275	-0.1657
0.05	-0.0311	-0.0659	-0.1002	-0.1318
0.09	-0.0212	-0.0436	-0.0668	-0.0892

Hence inview of equation (23) and above discussion it can be seen that for a given mean speed of the flow, the effective dispersion coefficeient D* decreases with increasing y_0 and ν while it increases with respect to μ.

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Chapter V

A Theoretical Analysis to the Effect of Wall Layer Thickness on Blood Rheology

Introduction :

Oka (1967) and Jones (1966) have explained the Coplay -Scott Blair result of the loss of apparent blood viscosity in a glass tube when the wall surface is coated with fibrin solution by introducing a slip velocity of the surface. Whitemore (1967) and Hyman (1973) have suspected the view and opine that slip at the solid surface has no evidence in practice and therefore is not a viable alternative to explain this reheological behaviour of blood. From the several studies it has been established that for blood flowing in narrow tubes (500 μm to 50 μm), a cell free plasma zone exists near the wall and the extent of this depends on many rheological parameters. From the analysis presented here, we see that apparent viscosity of blood decreases as the thickness of plasma layer increases. But there is no precise quantitative evidence that why this layer increases when its molecules come in contact with fibrinised surface of the wall. In one of his result Oba (1981) has contended that the thickness increases due to electric change on fibrin molecules.

Mathematical Analysis

Two fluids model of blood with a central core region of radius R and a peripheral layer of thickness $\delta = R - RO$, both satisfying casson equation

$$\tau^{1/2} = \tau_y^{1/2} + \eta^{1/2}\,\dot{\gamma}^{1/2} \qquad (1)$$

has been considered, where τ represents shear stress, η the casson viscosity, $\dot{\gamma}$ the strain rate, τ_y the yield stress and R the tube radius.

The equation of motion for steady incompressible laminar flow

$$\frac{\partial p}{\partial z} = \frac{1}{r}\frac{\partial}{\partial r}(r\,\tau_{rz}) \qquad (2)$$

When integrating with respect to the radius co-ordination r, and assuming that the stress at the axis be finite, gives

$$\tau_{rz} = \frac{\partial p}{\partial z}\frac{r}{2} \qquad (3)$$

Where p is the pressure, z the axial co-ordinate, τ_{rz} the shear stress normal to r in z direction. Substituting equation (3) in (1) and assuming that

$t_{rz(2)} = t_{rz(1)}$; $V_{(1)} = V_{(2)}$ at $r = R_0$

and $V_{(1)} = 0$ at $r = R$

the velocity in peripheral and core regions are obtained as

$$V_{(1)} = \frac{R\tau_R}{2\eta_1}\left[1-\frac{r^2}{R^2}-\frac{8}{3}\beta_{(1)}^{1/2}\left(1-\frac{r^{3/2}}{R^{3/2}}\right)+2\beta_{(1)}\left(1-\frac{r}{R}\right)\right] \qquad (4)$$

$$V_{(2)} = \frac{R\tau_R}{2\eta_2}\left[x^2\left(1-\frac{r^2}{R_0^2}\right)-\frac{8}{3}\beta_2^{1/2}x^{3/2}\left(1-\frac{r^{3/2}}{R_0^{3/2}}\right)+2\beta_2 x\left(1-\frac{r}{R_0}\right)\right] \qquad (5)$$

$$+\frac{R\tau_R}{2\eta_1}\left[(1-x^2)-\frac{8}{3}\beta_{(1)}^{1/2}(1-x^{3/2})+2\beta_1(1-x)\right]$$

Where, $x = 1-\frac{\delta}{R}$, $\beta_1 = \frac{\tau_{y(1)}}{\tau_R}$, $\beta_2 = \frac{\tau_{y2}}{\tau_R}$, τ_R is the shear stress at the wall and suffices (1) and (2) denote the values of the corresponding parameters in peripheral and core regions respectively.

Plug flow exists wherever the shear stress does not exceed yield stress and can be obtained by putting $r = \beta_2 R$ in equation (5). From (5) we may derive Jones result of one fluid model of blood flow.

We calculate apparent viscosity η_a in the form as

$$\frac{1}{\eta_a} = \frac{1}{\eta_2}\left[1-\frac{16}{7}\beta_2^{1/2}+\frac{4}{3}\beta_2-\frac{1}{21}\beta_2^4\right]+\frac{4\delta}{R}\left[\frac{1+\beta_1-2\beta_1^{1/2}}{\eta_1}-\frac{1+\beta_2-\beta_2^{1/2}}{\eta_2}\right] \qquad (6)$$

We observe that the loss in η_a is caused due to increase in δ (as $\frac{\eta_2}{\eta_1} > \frac{1 + \beta_2 - 2\beta_2^{1/2}}{1 + \beta_1 - 2\beta_1^{1/2}}$)

Results are discussed in the form of following two cases

Case I. When $\beta_1 = 0$ and the relation $\eta_1 = \eta_2 (1 - \alpha \phi)$ holds, where ϕ is hematocrit function and α is experimental constant. Then

$$\frac{1}{\eta_a} = \frac{1}{\eta_1}(1 - \alpha\phi)\left[1 - \frac{16}{7}\beta_2^{1/2} + \frac{4}{3}\beta_2 - \frac{1}{21}\beta_2^4\right] \qquad (7)$$

$$+ \frac{1}{\eta_1}\left[(1 - \alpha\phi)(2\beta_2^{1/2} - \beta_2) + \alpha\phi\right]\frac{4\delta}{R}$$

The above equation (7) shows that for any desired concentration ϕ, η_a decreases with δ.

Case II. When $\beta_1 = 0 = \beta_2$

In this case the above theory gives the result for Newtonian model in which the fluids of different viscosities η_1 and η_2 flow in both regions but have the same constitutive stress-strain relation of Newtonian structure. The apparent viscosity η_a is calculated as

$$\frac{1}{\eta_a} = \frac{1}{\eta_1}\left[1 - (1 - \frac{4\delta}{R})\alpha\phi\right] \qquad (8)$$

Equation (8) shows that for $\delta = R/4$, $\eta_a = \eta_1$ and when $\delta > R/4$ then η_a decreases with increasing δ.

This supports the copley second result in which the experiment has been performed for plasma fluid alone.

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CHAPTER VI

Theoretical Model of Steady Blood Flow in Narrow Vessel

Introduction :

Mathematical modelling of blood flow in different vessels under various physiological conditions have drawn the attention of several workers. Since poiseuilles empirical euqation of finding out the apparent viscosity of viscous fluid in fine glass capillaries, many workers attempted to find the apparent viscosity invivo and invitro, which is an important parameter affecting the blood rheology. Fahraeus and Lindquist found that apparent viscosity of blood decreases with the decrease of tube diameter below 500 μm down to 7 μm. Recently, Kiani and Hudetz (1991) developed the model assuming no yield stress behaviour of blood in these vessels. It has been recognised by several experimental observations and viscometric datas that blood possesses considerable amount of yield stress in narrow vessels. Scott Blair (1959), Charm and Kurland (1968), bugliarello and Sevilla (1970), Oba (1981, 89) have proposed casson constitutive equation to analyse the blood behaviour in narrow vessels at low and high shear rates. In the present analysis we have assumed that the fluid in core region satisfy casson equation and in marginal layer region satisfy Newtonian equation. Apparent viscosity of blood has been determined as a function of yield stress, vessel diameter and peripheral layer (PPL) thickness.

Mathamatical Analysis :

Incompressible fully developed laminar flow of blood is considered in a circular tube of small diameter. Two-layer flow model is assumed. Core fluid follow the casson equation.

$$\tau^{1/2} = \tau_y^{1/2} + \mu^{1/2}\ \dot{\gamma}^{1/2} \qquad (1)$$

and plasma peripheral layer follows the Newtonian equation

$$\tau = \mu_p\ \dot{\gamma} \qquad (2)$$

Where τ is shear stress, τ_y the yield stress, $\dot{\gamma}$ the shear rates, μ_c casson core viscosity, μ_p Newtonian plasma viscosity.

Steady incompressible laminar flow equation

$$\frac{\partial P}{\partial Z} = \frac{1}{r}\frac{\partial}{\partial r}(r\ \tau_{rz}) \qquad (3)$$

after integration yields

$$\tau_{rz} = \frac{pr}{2L} \qquad (4)$$

Where

z, r = axial and radial measurements

τ_{rz} = shear stress normal to r in z direction

p = pressure drop along length L of the tube.

At the interfacial boundary between core and marginal fluids, shear stresses are equal and at the wall surface noslip condition are assumed.

Substituting (1) and (2) in (4) we get

$$V_p = \frac{P}{4L\ \mu_p}(Rw^2 - r^2); \quad R_c \le r \le Rw \qquad (5)$$

$$V_c = \frac{P}{4L\mu_c}(R_c^2 - r^2) - \frac{4}{3}\left(\frac{P\tau y}{2L}\right)^{1/2}\frac{1}{\mu_c}(R_c^{3/2} - r^{3/2})$$

$$+ \frac{\tau_y}{\mu_c}(R_c - r) + \frac{P}{4L\mu_p}(R_w^2 - R_c^2) \quad ; 0 \le r \le Rc \qquad (6)$$

Where V_p denotes plasma velocity and Vc denotes core velocity.

Plug velocity is obtained by putting $r=R_c$ in equation (6) as

$$V_1 = -\frac{P}{4L\mu_c}(R_c^2 - R_1^2) - \frac{4}{3\mu_c}\left(\frac{P\tau_y}{2L}\right)^{1/2}(R_1^{3/2} - R_1^{3/2}) + \frac{\tau_y}{\mu_c}(R_c - R_1)$$

$$+ \frac{P}{4L\mu_p}(R_w^2 - R_c^2) \qquad (7)$$

The total flow rate $Q = Q_p + Q_c + Q_1$

or $$Q = \int_{R_c}^{R} 2\pi r dr \cdot V_p + \int_{R_1}^{R_c} 2\pi r dr \cdot V_c + \pi R_1^2 \cdot V_1$$

or $$Q = \frac{P\pi R_w^4}{8L\mu_p}\left[1-\left(\frac{R_c}{R_w}\right)^4 + \left(\frac{\mu_p}{\mu_c}\right)\left\{\frac{R_c^4}{R_w^4} - \frac{16}{7}\left(\frac{R_1}{R_w}\right)^{7/2}\cdot\left(\frac{R_1}{R_w}\right)^{1/2}\right.\right.$$

$$\left.\left. + \frac{4}{3}\left(\frac{R_c}{R_w}\right)^3\left(\frac{R_c}{R_w}\right) - \frac{1}{21}\left(\frac{R_c}{R_w}\right)^4\right\}\right]$$

Where plug radius $R_1 = \frac{2L\tau_y}{P}$ (8)

The apparent blood viscosity μ_{app} is obtained as

$$\mu_{app} = \mu_p\left[\left\{1-\left(1-\frac{2\delta}{d}\right)^4\right\} + \frac{\mu_p}{\mu_c}\left\{\left(1-\frac{2\delta}{d}\right)^4 - \frac{16}{7}\left(1-\frac{2\delta}{d}\right)^{7/2}\cdot\alpha^{1/2}\right.\right.$$

$$\left.\left. + \frac{4}{3}\left(1-\frac{2\delta}{d}\right)^3\alpha - \frac{1}{21}\alpha^4\right\}\right]^{-1} \qquad (9)$$

Where

$$\alpha = \frac{R_1}{R_w}, \quad \delta = R_w - R_c, \quad d = 2R_w$$

Upto first order smallness of δ, equation (9) reduces to

$$\mu_{app} = \mu_p\left[\frac{\mu_p}{\mu_c}F(\alpha) + \frac{8\delta}{d}\left(1 - \frac{\mu_p}{\mu_c}\phi(\alpha)\right)\right]^{-1} \qquad (10)$$

Where $$F(\alpha) = 1 - \frac{16}{7}\alpha^{1/2} + \frac{4}{3}\alpha - \frac{1}{21}\alpha^4$$

$$\phi(\alpha) = 1 - 2\alpha^{1/2} + \alpha$$

As α is a measurement of yield stress, therefore at high shear rates it takes small value but not zero as chosen by Kiani and Hudetz (1991). For

constant $F(\alpha)$ and $\phi(\alpha)$, apparent viscosity decreases with the increase in δ. Fahraeus Lindquist effect can be observed from equation (10). Charm and Kurland have shown that δ increases with decreasing α. We have taken the datas of Charm and Kurland and shown the variation of μ_{app} with α at constant $\frac{\delta}{d}$ in table I. Assuming that the casson viscosity is related by general equation

$$\mu_c = \mu_p (1-KH)^{-1} \qquad (11)$$

Where H is a function of hematocrit and K is an experimental constant, we can obtain μ_{app} at different concentration of hematocrit.

TABLE 9

[Charm and Kurland (1968), μ_p = .0125 poise, μ_c = .0259 pose, $\frac{\delta}{d}$ = .01, H = .448]

μ_{app}	α	$F(\alpha)$	$f(\alpha)$
2.7790	.0005	.9495	.9559
2.8293	.0010	.9291	.9378
3.0584	.0050	.8451	.8636

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